Lattice Gauge Theory in the Proton Driver Era

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Outline

- Idiosyncratic lightning review of lattice QCD
- f_π, f_K and light quark masses
- Homework for Yuval, Uli, *etc.*
- Assumptions for future
- Projections (unless, as I hope, time runs out)

Lattice QCD

A Multi-Scale Problem

- QCD is a multi-scale problem
  - Λ: the characteristic scale of the strong interaction
  - m_q: light quark masses $m_q \ll \Lambda$: good for u, d, (s)
  - m_Q: heavy quark masses $m_Q \gg \Lambda$: good for t, b, (c)
  - a^{-1}: ultraviolet cutoff, always needed in QFT
  - L^{-1}: infrared cutoff, often helpful in QFT

- Ken Wilson said, integrate the functional integral numerically (with finesse and brute force):

$$\int \mathcal{D}A\, \mathcal{D}\psi \mathcal{D}\bar\psi\; \bar\psi_u \gamma_5 \psi_d(x) \bar\psi_d \gamma_5 \psi_u(y)\; e^{-S_g - \bar\psi M \psi} = \int \mathcal{D}A\; \mathrm{tr}[G_d(x,y)\gamma_5 G_u(y,x)\gamma_5]\, \det M\; e^{-S_g}$$

$M = [\not{D} + m]_{\mathrm{lat}}$ $\qquad$ S_g = lattice gauge action

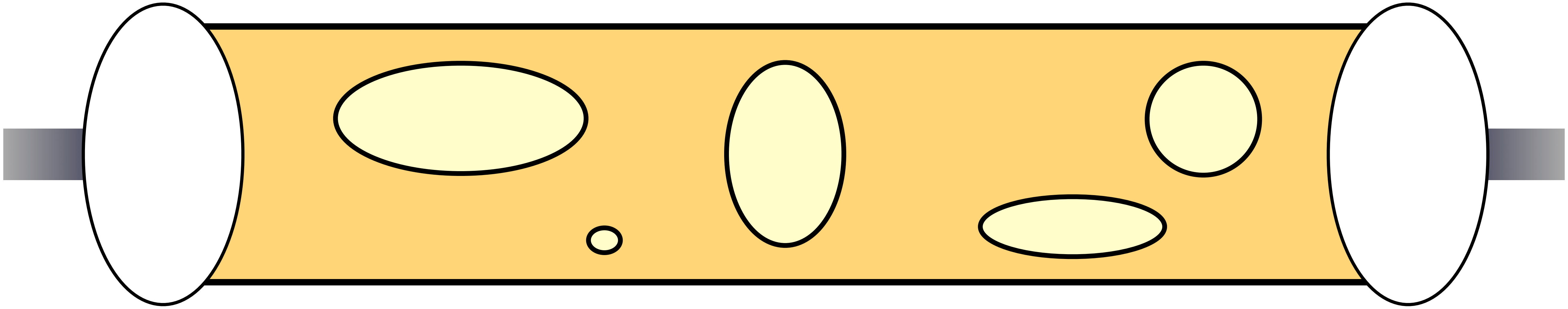
- $G = M^{-1}$ (quark propagators): expensive
- $\det M$ (sea quark loops): very expensive

Systematics

- MC treats Λ exactly, up to statistical errors.
- Control systematics with effective field theories:
  - $m_q = rm_s > m_d$: chiral perturbation theory (χPT)
  - $L < \infty$: general EFT of hadrons; χPT
  - $a \neq 0$: Symanzik effective field theory
  - $m_Q a \sim 1$: HQET, NRQCD [hep-lat/0310063]
- verify with numerical data, then extrapolate

Quenched Approximation

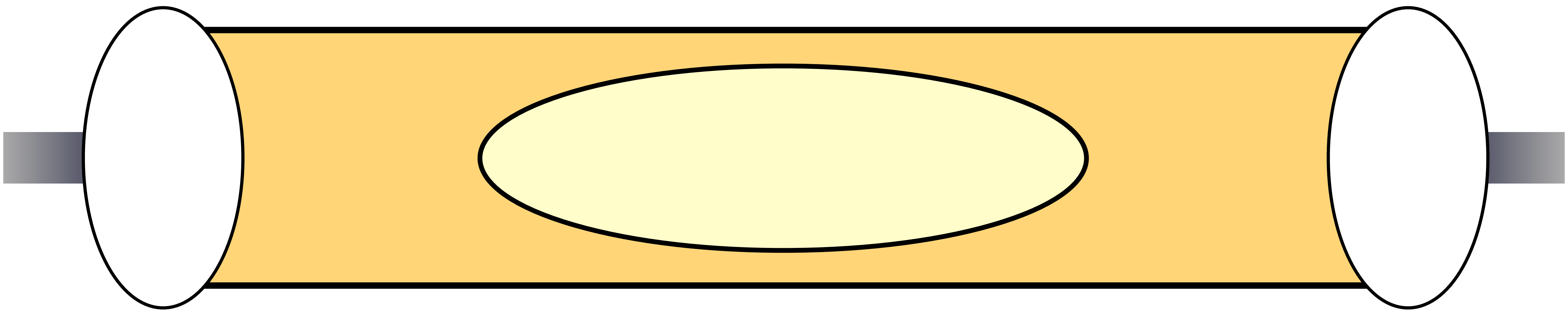
Full QCD has (expensive) quark loops.



- Replace det M with 1, and compensate by shifting bare gauge coupling and bare masses. "Dielectric".
- Arguably OK if all light quarks had mass $m_q \sim \Lambda$.
- The Main Ring era: not even the Main Injector era.

Chiral Extrapolation

Virtual quark loops: $B \to \left\{ \begin{array}{c} B^*\pi \\ B_s^*K \\ B_{(s)}^*\eta \end{array} \right\} \to B.$



- Loops yield non-analytic behavior, *e.g.*, $m_\pi^2 \ln m_\pi^2$.
- Extrapolation needs small enough m_q.

Lattice Fermions

- Naïve: 16 species per field, called "tastes".
- Wilson: 1 taste (flavor), but hard chiral symmetry breaking ⇒ fine tuning ⇒ $m_q > 0.7 m_s$ [JLQCD, QCDSF, ...]. Twisted mass helps, but new.
- Staggered: still 4 tastes per field, but remnant of chiral symmetry ⇒ $m_q > 0.15 m_s$ [MILC].
- Ginsparg-Wilson (domain wall or overlap): flavor simple, full chiral symmetry. More expensive—but relevant to future *K* calculations.

The Berlin Wall

$$\texttt{cost} \propto \left(\frac{m_{\rm V}^2}{m_{\rm PS}^2}\right)^3 L^{4+1}\ a^{-(4+3)}$$

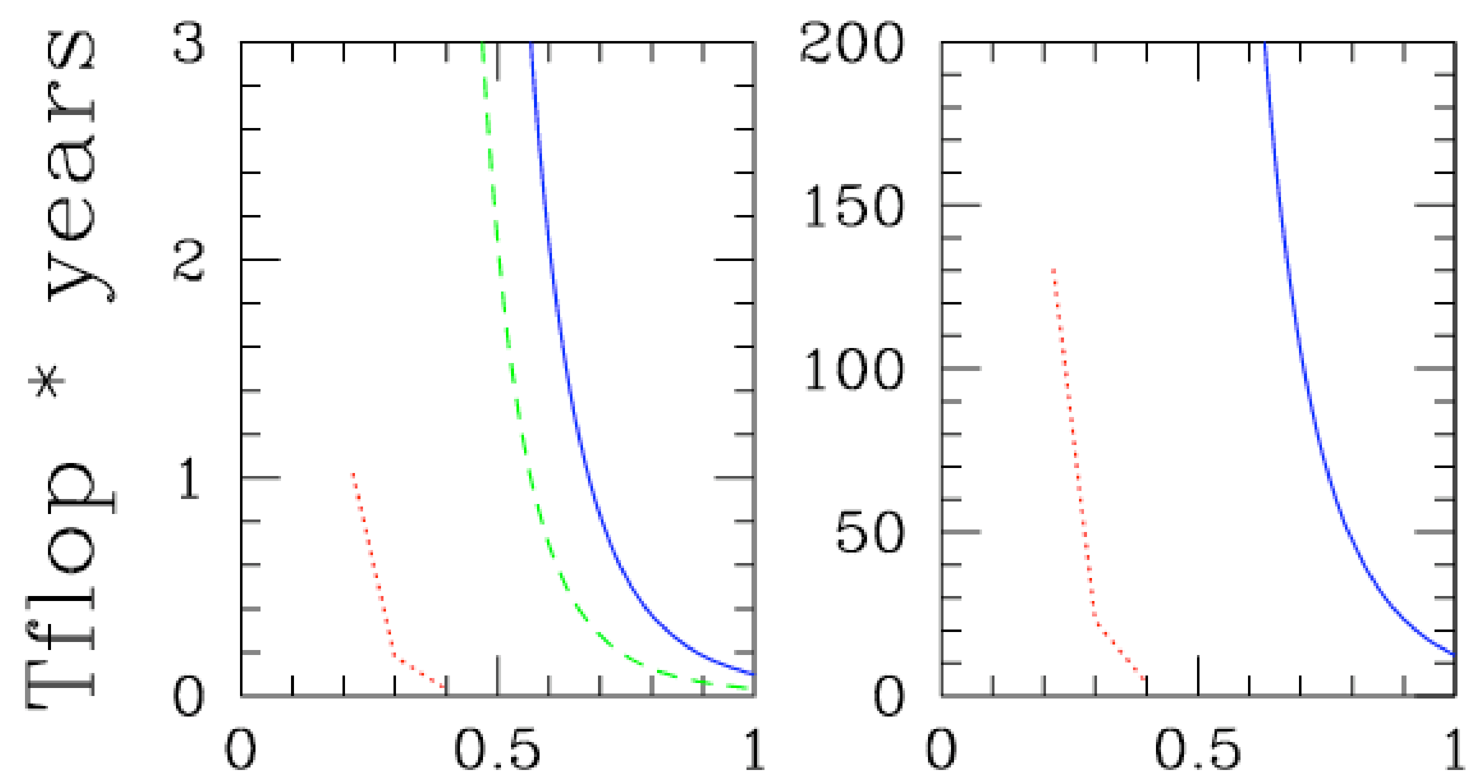
- cost for Wilson

  $\equiv$ 3 times faster

- cost for staggered

- Plot from Jansen, who had input from Ukawa & Gottlieb

hep-lat/0311039



$m_{\rm PS}/m_{\rm V}$

a = 1/11 fm measured

$m_{\rm PS}/m_{\rm V}$

a = 1/22 fm extrapolated

Staggered Quarks

- Staggered fermions have always been fast.
- Discretization effects $O(a^2)$, but "large".
- Traced to "taste-changing" interactions.
- Systematically removed by Orginos & Toussaint:
  - the "Fat7 action"
- Remaining $O(a^2)$ removed by Lepage
  - the "asqtad action": $O(\alpha_s a^2)$, $O(a^4)$ and "small".

Gold-plated Quantities

- Some quantities are under much better control:
  - 1 hadron in the initial state & 0 or 1 in the final state;
  - stable, or narrow and not too close to threshold.
- *Chiral extrapolation must also be under control!*
- D^*, ϕ, … not gold-plated, but perhaps not bad. η'?
- (almost) elastic ρ, Δ, $K \to \pi\pi$ much, much harder.
- No experience with $\langle H | T\, O_1(x)\, O_2(0) | H \rangle$

Unquenched vs. Quenched

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The MILC Ensembles

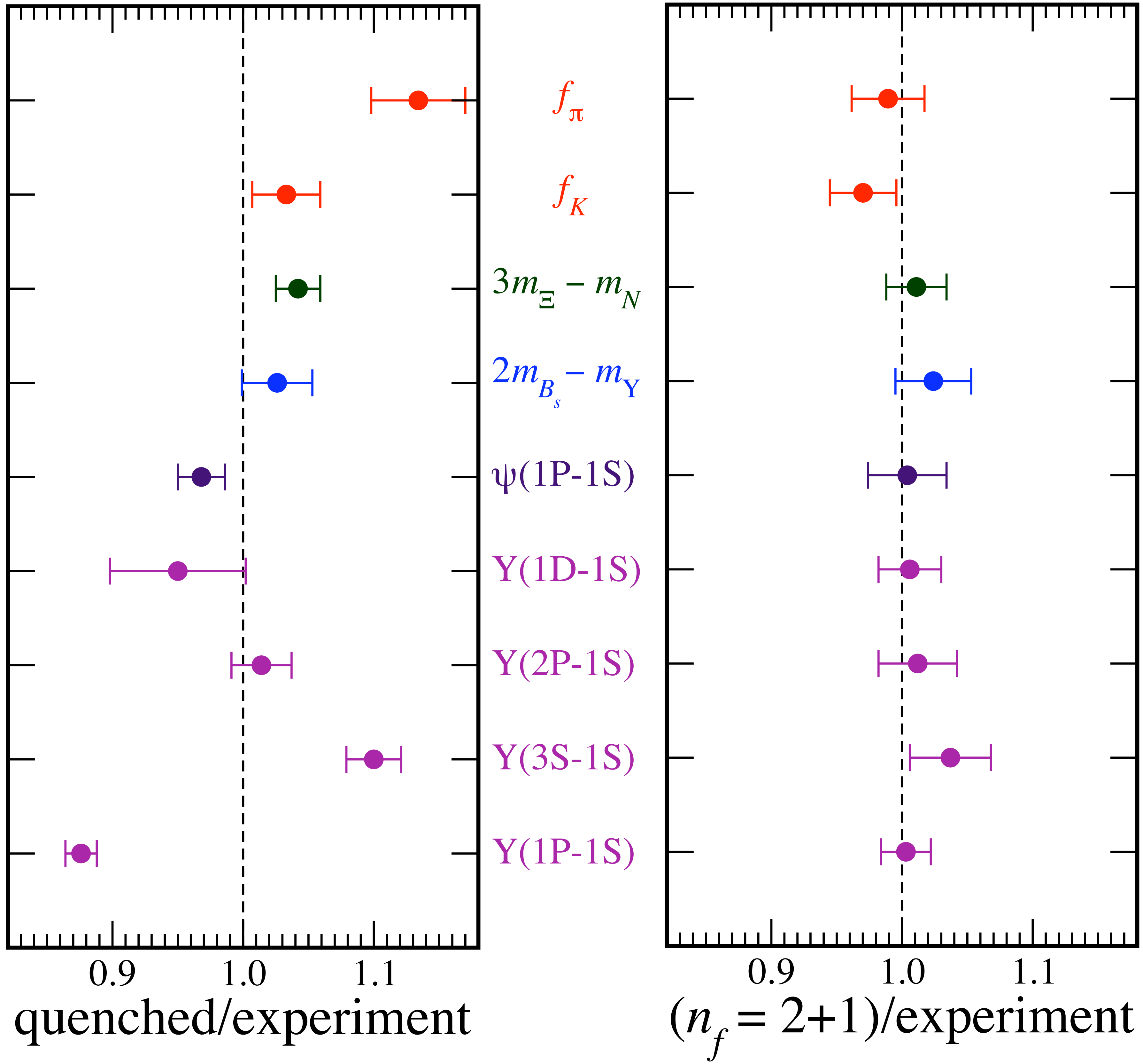
- MILC Collaboration = dozen or so physicists at Arizona, UCSB, Colorado, FSU-SCRI/APS, Indiana, Pacific, Utah, Washington U. (St. Louis)
- Improved staggered quarks (asqtad action)
- Sea quark loops (det M) for 2 + 1 flavors
- a = 1/8, 1/11 fm (also 1/6 fm, but omitted)
- Many (valence and sea) m_q down to $0.15 m_s$
- Several hundred lattice gauge fields per ensemble

- Freely available over the internet.
- Several groups started looking at light hadrons (MILC), hadrons with bottom quarks (HPQCD), hadrons with charmed quarks (Fermilab).
- All of the QCD scale was being probed.
- A consistent picture emerged: after tuning $1 + n_f$ parameters, we checked 9 other mass splittings and decay constants.

Tune Bare Couplings

- pick $g_0^2(a)$ and use Δm_Υ(2S-1S) to deduce a
  - not very sensitive to quark masses, even m_b
- light (u, d) and strange masses tuned to (m_π^2, m_K^2)
- charmed mass tuned to (spin-averaged) m_{Ds}
- bottom mass tuned to m_Υ(1S)
- Useful to compare quenched vs. unquenched.

Davies et al., hep-lat/0304004



- Because staggered quarks come in four tastes, we have used $[\det_4 M]^{1/4}$ for $\det_1(\not{D} + m)$.
- $\det_4 M^{1/4}$ looks non-local and, hence, terrifying.
- However:
  - Correct in perturbation theory.
  - Chiral anomalies incorporated correctly.
  - Long-distances well described by a version of χPT designed to handle it.
- “Not proven,” but several positive indications.

New Investigations

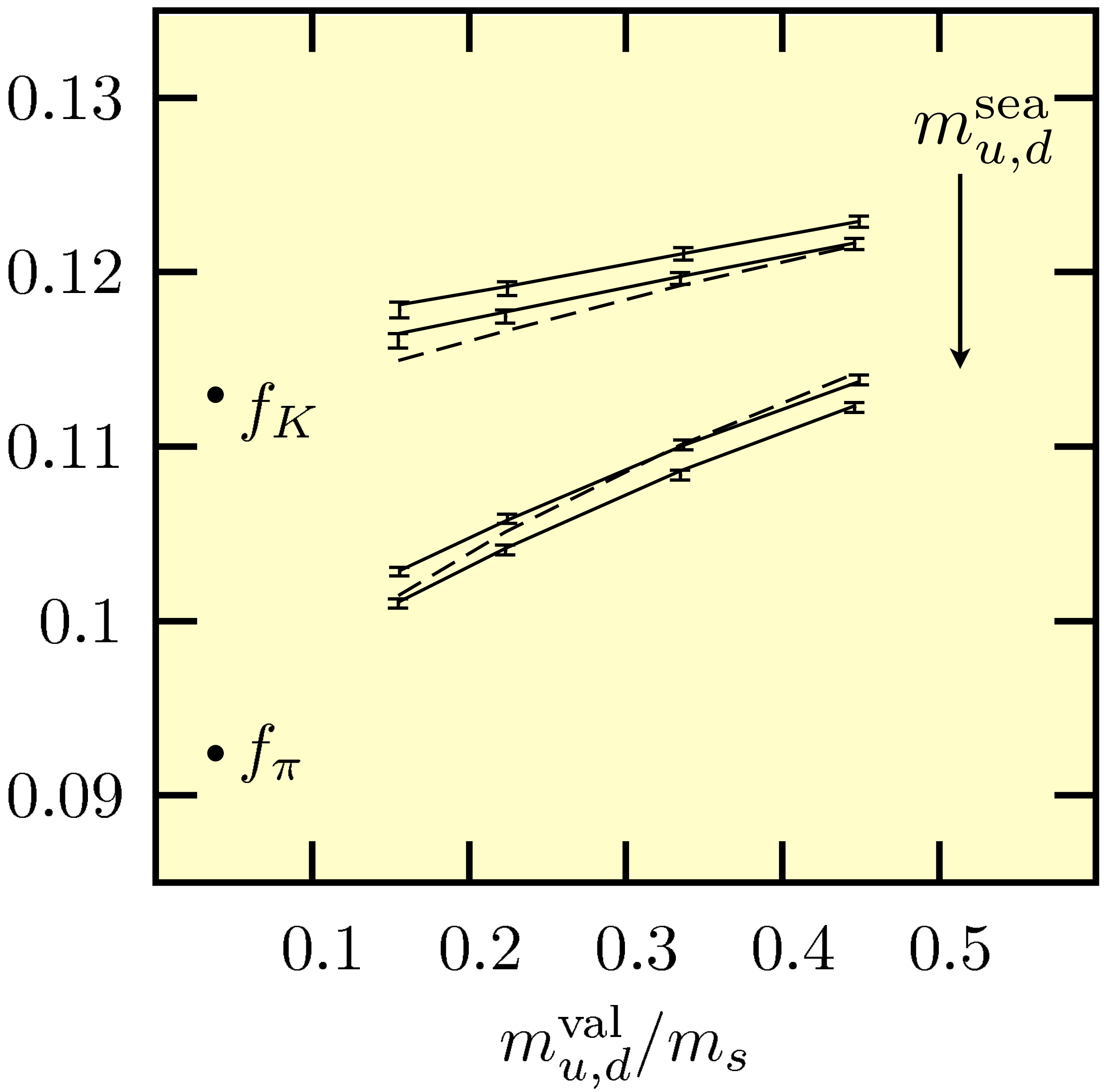
- Adams
  - rigorous mathematical proof in a *related* context
- Davies, Follana, & Hart; Dürr & Hölbling
  - 4-fold degeneracy of eigenvalues emerge; topology
- Bunk *et al.*
  - $M^{1/4}$ is non-local, but don't know about $[\det_4 M]^{1/4}$
- Neuberger
  - 6-d framework to test locality

Summary So Far

- Lattice QCD with improved staggered quarks agrees with Nature for 5+9 gold-plated quantities.
- Only improved staggered fermions can achieve the following (in the near term):
  - 2+1 flavors of sea quark
  - light enough quarks for chiral perturbation theory
- Very promising for B, D, K, and π physics.

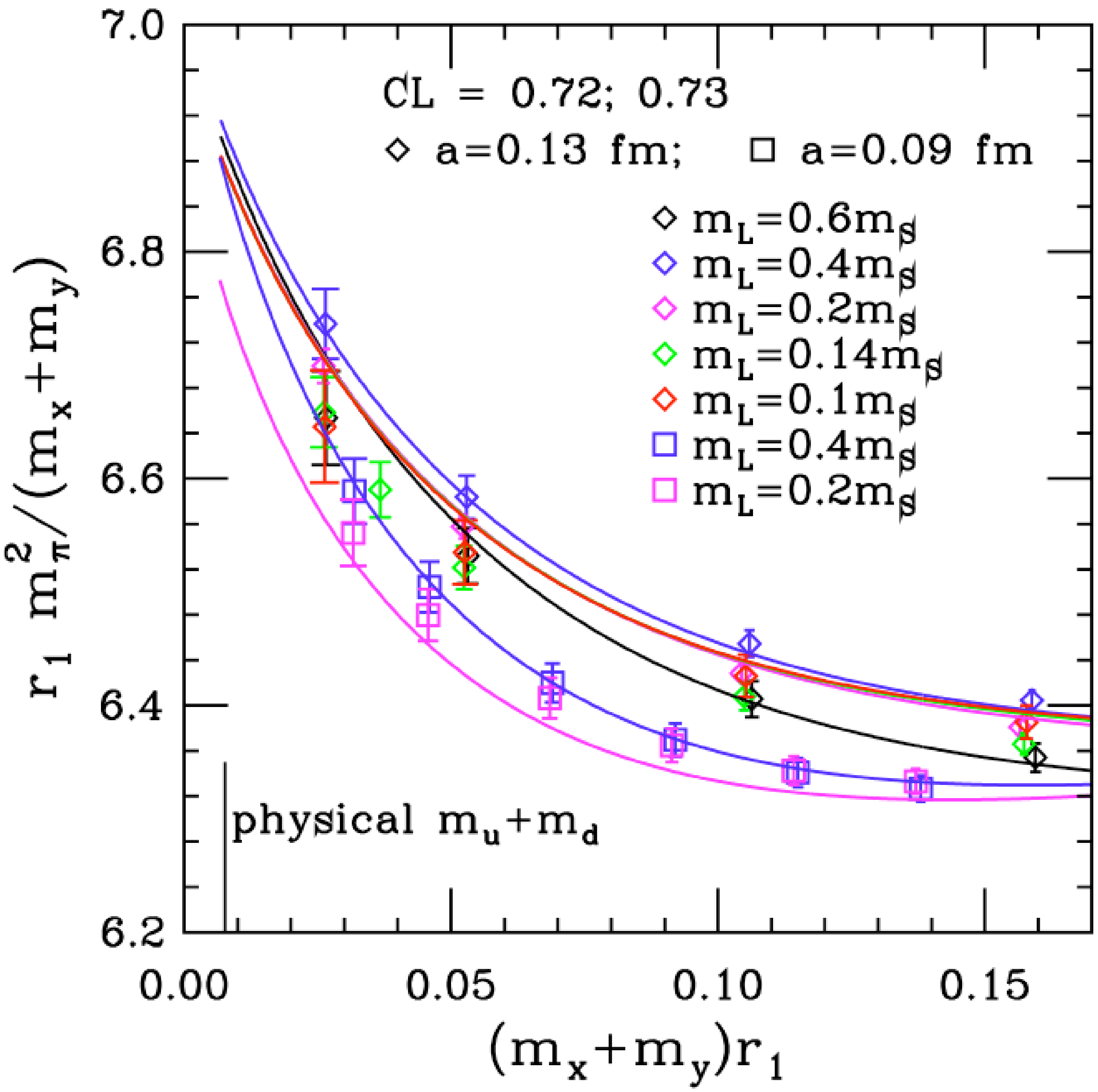
f_π and f_K

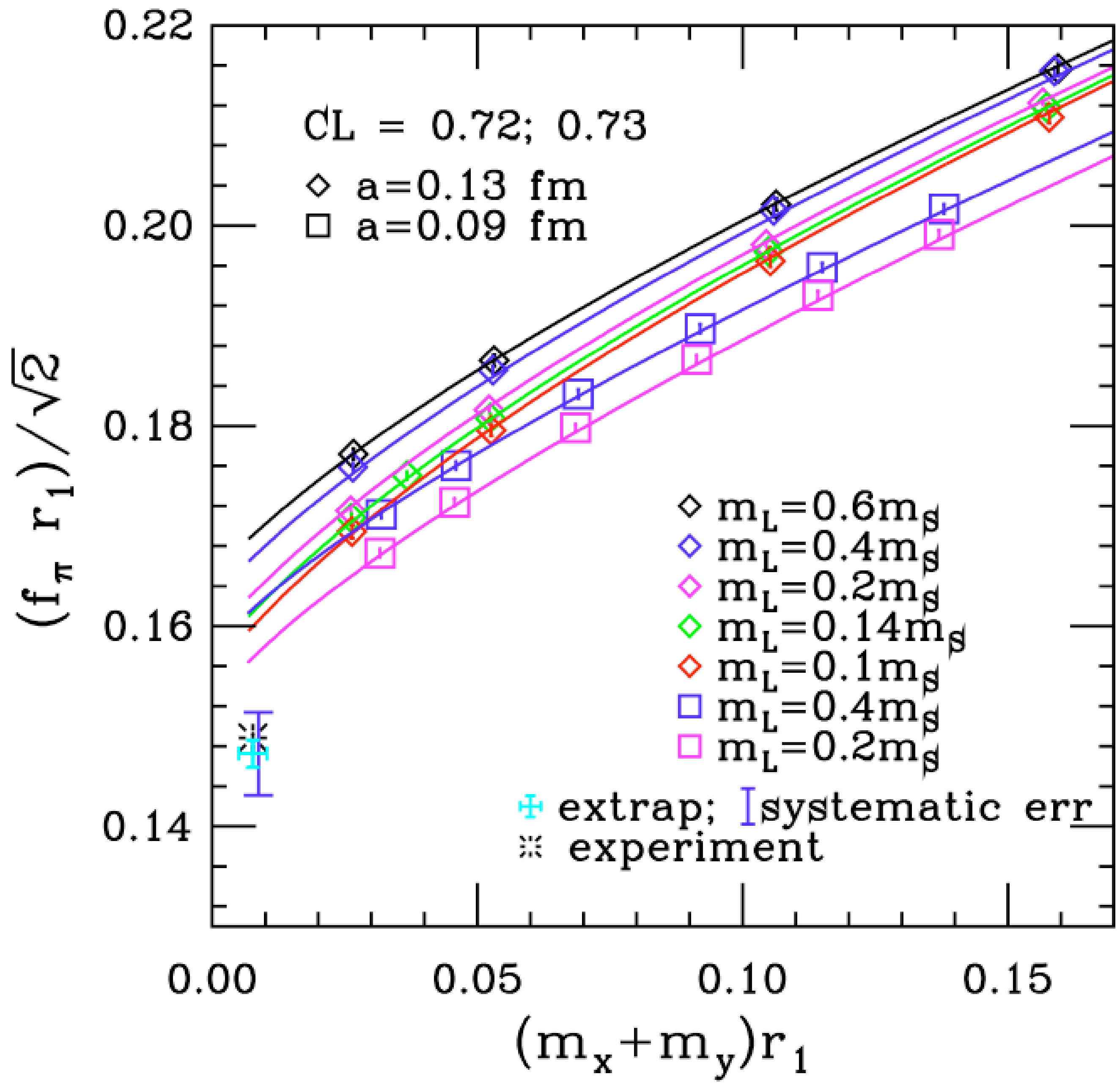
Chiral Extrapolation



- Dots at 0.04 are experimental.
- Error bars are lattice QCD.
- Linear extrap (by eye).
- Gasser-Leutwyler χlog gets closer (solid).
- Sharpe-Shoresh χlog even closer (dashed).

- Finally, χPT can be modified to incorporate the 4 tastes and the 1/4 root [Aubin & Bernard].





One fit to all quark mass combos & both lattice spacings!

- Four extrapolations:
  - linear
  - continuum χPT, assuming $m_q^{\mathrm{val}} = m_q^{\mathrm{sea}}$
  - continuum χPT, with $m_q^{\mathrm{val}} \neq m_q^{\mathrm{sea}}$
  - χPT with taste-symmetry breaking and ....
- Successively more accurate.
- Hard to reconcile with a non-local underlying theory.

Results

- $f_\pi = 129.5(0.9)(3.4)(0.0)$ MeV hep-lat/0407028
- $f_K = 156.6(1.0)(3.5)(0.1)$ MeV hep-lat/0407028
- $f_K/f_\pi = 1.210(4)(13)(1)$ hep-lat/0407028
- $m_s(2\text{ GeV}) = 76(0)(3)(0)(7)$ MeV hep-lat/0405022
- $2m_s/(m_u + m_d) = 27.4(1)(4)(1)$ hep-lat/0405022
- $m_u/m_d = 0.43(0)(1)(8)$ hep-lat/0407028
- statistics, stagχPT, EM, matching

Homework for Yuval, Uli, …

Classify QCD

- Gold-plated & silver-plated matrix elements
  - fundamental QCD parameters: α_s, m_q
- Much harder: (nearly) elastic decays, *e.g.*, $K \to \pi\pi$
- Ideas needed (QCD + QED + clever sources?)
  - hadronic light-by-light for (g–2)
  - $K \to \pi\gamma^*\gamma^*$ for $K \to \pi\mu\mu$
- Impossible, *e.g.*, $B \to \pi\pi$ (because inelastic)

Assumptions for the Proton Driver Era

- When: 6-12 years hence $\Rightarrow 2^4$–$2^8 \times$ better CPU
  - perhaps more factors of 2 for better funding & ideas
- Assume basic paradigm remains: Monte Carlo + chiral perturbation theory.
- Worst case: staggered fermions are found to have a fatal flaw. Then we will use CPU to get back to few-% errors with other fermion methods.
- Best case: can reduce statistical errors by ÷ 10
  - assume systematics scale
- Assume matching improves (where needed).

Semileptonic Projections

Two Targets

- $|V_{ud}|_{\mathrm{PDG}} = 0.9738(0)(5) \quad \rightarrow (?)(2)$
- $|V_{us}|_{\mathrm{KTeV}} = 0.2252(8)(26) \rightarrow (6)(6)$
  - via leptonic decays, MILC á la Marciano [hep-ph/0402299: f_K/f_π + PDG $|V_{ud}|$] find $|V_{us}| = 0.2219(26)$
  - if errors go down by a factor of five, target is reached
- Semileptonic decays perhaps more promising

$D \to \pi, K; B \to \pi, D$

Okamoto et al., hep-lat/0409116

CKM matrix with $n_f = 3$ LQCD
(preliminary)

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \text{N/A} & \text{N/A} & 3.6(5)(4)(3)\times 10^{-3} \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ 0.24(1)(2)(2) & 0.97(4)(8)(2) & 3.8(1)(1)(6)\times 10^{-2} \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \text{N/A} & \text{N/A} & \text{N/A} \end{pmatrix}$$

Experimental errors

4/9 being determined with $n_f = 3$ LQCD.

LQCD unitarity check!

$$(|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(4)(8)(2)$$

slide from talk at Lattice 2004

- $B \to D$ better (1.5% systematic instead of 10%) because of zero-recoil double-ratio

$$|h_+^{B\to D}|^2 = \frac{\langle B|\bar{b}\gamma_4 c|D\rangle\langle D|\bar{c}\gamma_4 b|B\rangle}{\langle B|\bar{b}\gamma_4 b|B\rangle\langle D|\bar{c}\gamma_4 c|D\rangle}$$

- heavy-quark symmetry says form factor is ≈ 1
- double ratio ensures that errors scale as $h_+ - 1$
  - ≡ 1.5% is actually 17% of a 7.5% deviation
- works with any symmetry, *e.g.*, isospin, SU(3)
- $K \to \pi$, $n \to p$, $\pi \to \pi$, $K \to K$

$K \to \pi$

- Now the precise zero-recoil double ratio is

$$|f_0^{K\to D}(q^2_{\mathrm{max}})|^2 = \frac{\langle K|\bar{s}\gamma_4 u|\pi\rangle\langle\pi|\bar{u}\gamma_4 s|K\rangle}{\langle K|\bar{s}\gamma_4 s|K\rangle\langle\pi|\bar{u}\gamma_4 u|\pi\rangle}$$

  APE has ‰ precision in quenched QCD

- But one needs $f_+(0) = f_0(0)$
  - APE calculates f'_+ and $f'_+ - f'_0$ with ~20% precision
- $f(0) = 1 + 0_{\mathrm{AG}} + \mathrm{known}_{\chi\mathrm{PT}} + O\left(\left(\frac{m_K^2 - m_\pi^2}{8\pi^2 f_\pi^2}\right)^2\right)$
- About 3%; Leutwyler & Roos and APE find −4%

$\pi^+ \rightarrow \pi^0$

- Same methods would apply
- q^2 extrapolations negligible
- Error is a fraction of $\left(\frac{m^2_{\pi^+} - m^2_{\pi^0}}{8\pi^2 f^2_\pi}\right)^2 \sim 10^{-6}$
- But need to worry about
  - isospin breaking, also in sea quarks (for π^0-η mixing)
  - structure-dependent radiative corrections

$K^0 \to K^+$

- $\mathrm{BR}(K_S) \sim 10^{-11}$; $\mathrm{BR}(K_L) \sim 5 \times 10^{-9}$
- q^2 extrapolations again negligible
- Error is a fraction of $\left(\frac{m^2_{K^0} - m^2_{K^+}}{8\pi^2 f^2_\pi}\right)^2 \sim 10^{-5}$
- But need to worry about
  - isospin breaking, but only for valence quarks
  - structure-dependent radiative corrections

Chiral Perturbation Theory

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- In many cases the matrix element *you* want is one that, to lattice QCD, is not gold-plated.
- Perhaps χPT can be used more aggressively.
  - use gold-plated quantities + lattice QCD to determine chiral parameters [see MILC's]
  - use χPT for *your* phenomenology
  - plug in lattice-derived chiral parameters

ε'/ε and $\Delta I = 1/2$ Rule

Baryons

Baryons in LatQCD

- Baryons always have larger statistical errors than (pseudoscalar) mesons
- ∴ crosschecks of quark masses, CKM, ...
  - *e.g.*, m_s from MILC-HPQCD yields m_Ω within 0.5σ
  - $|V_{us}|$ from hyperon decay less precise than from K_{l3}
- moments of *pdf*s should be gold-plated
- nucleon decay constant

Tests

D Decays

- CKM drops out of

$$\frac{1}{\Gamma_{D_s \to l\nu}} \frac{d\Gamma_{D \to K l\nu}}{dE_K} \propto \left| \frac{f_+^{D \to K}(E_K)}{f_{D_s}} \right|^2$$

$$\frac{1}{\Gamma_{D \to l\nu}} \frac{d\Gamma_{D \to \pi l\nu}}{dE_\pi} \propto \left| \frac{f_+^{D \to \pi}(E_\pi)}{f_D} \right|^2$$

- Pure tests of non-perturbative QCD $\Leftarrow$ CLEO-*c*
- Functions of energy
- Similarly for K decays

B_c from [Glasgow/Fermilab]

- with quarkonium baseline (preliminary)
  - $m_{B_c} = 6.307 \pm 0.002^{+0.000}_{-0.010}$ GeV
  - systematic dominated by the B_c Darwin correction
- with heavy-light baseline (preliminary)
  - $m_{B_c} = 6.253 \pm 0.017^{+0.030\sim 50}_{-0.000}$ GeV
  - systematic dominated by the D_s Darwin correction
- DØ and CDF will reduce error 400 → 10s MeV